The sharp constants in the real anisotropic Littlewood's 4/3 inequality

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Abstract

The real anisotropic Littlewood's 4/3 inequality is an extension of a famous result obtained in 1930 by J. E. Littlewood. It asserts that, for $a, b \in (0, \infty)$, the following conditions are equivalent:

- There is an optimal constant $\mathsf{L}^{\mathbb{R}}_{a,b} \in [1, \infty)$ such that

$$\left(\sum_{k=1}^{\infty} \left(\sum_{j=1}^{\infty} \left| A\big(\mathbf{e}^{(k)}, \mathbf{e}^{(j)}\big) \right|^{a} \right)^{\frac{b}{a}} \right)^{\frac{1}{b}} \leq \mathsf{L}^{\mathbb{R}}_{a,b} \cdot \|A\|$$

  for every continuous bilinear form $A\colon c_0 \times c_0 \to \mathbb{R}$.

- The values a, b satisfy $a, b \geq 1$ and $\frac{1}{a} + \frac{1}{b} \leq \frac{3}{2}$.

Several authors have obtained the values of $\mathsf{L}^{\mathbb{R}}_{a,b}$ for diverse pairs (a, b). In this talk I will show how to obtain the complete list of such optimal values, as well as new estimates for $\mathsf{L}^{\mathbb{C}}_{a,b}$ (the analog for continuous $\mathbb{C}$-bilinear forms), which are exact in several cases. If time permits, I will sketch the proof of a variant of Khinchin's inequality for Steinhaus variables, which involves the values $\mathsf{L}^{\mathbb{C}}_{1,r}$, as well as some estimates for the (q, s)-cotype constants of the spaces $\ell_1(\mathbb{K})$ (with $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$) in terms of the values $\mathsf{L}^{\mathbb{R}}_{1,q}$.

This talk is based on joint work with N. Caro Montoya and D. Serrano-Rodríguez, featured in [1].

References

[1] Caro-Montoya, N. & Núñez-Alarcón, D. & Serrano-Rodríguez, D. - *The sharp constants in the real anisotropic Littlewood's* 4/3 *inequality*, Proc. Amer. Math. Soc., **153**, (2025). https://www.ams.org/journals/proc/0000-000-00/S0002-9939-2025-17367-1/?active=current